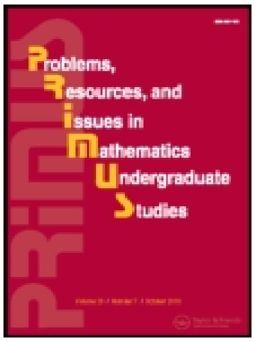

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Keychain Ziplines: A Practical Way to Study Velocity in the Calculus Classroom

Lisa Driskell and Audrey Malagon

Abstract: Using materials that are easy to procure, students create a zipline for a weighted keychain. They measure distances and times to calculate average velocities and explore the idea of instantaneous velocity. This inquiry-based calculus activity has been tested as a first day activity in classrooms at a variety of institutions and has received positive student and instructor feedback.

Keywords: Zipline, calculus, velocity, tactile learning, inquiry-based learning, average velocity, instantaneous velocity.

1. INTRODUCTION

Perhaps no topic lends itself better to a hands-on demonstration than the idea of average and instantaneous velocity. The trouble, however, is that common examples such as objects in free fall or balls rolling down ramps are hard to reproduce in the classroom in ways that allow for accurate student measurement. Our keychain zipline activity provides an easy-to-build example of a moving object for which students can accurately record time, distance, and velocity measurements (Figure 1). It introduces students to the idea of instantaneous velocity and sets up a framework for the discussion of limits. When used as a first day activity in Calculus I, it allows students to meet each other and sets the tone for an active class. This activity has been tested in classes ranging from 15 to 35 students and in public university and private liberal arts college settings.

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Figure 1. A keychain moves down the zipline as students work together to take time measurements using a stopwatch on a phone (color figure available online).

2. OBJECTIVES

The objectives for this activity are twofold: learning objectives in calculus and goals for classroom dynamics. Regarding calculus, we want to review average velocity calculations, introduce the concept of instantaneous velocity, and have students explore the relationship between the two as we set up a framework for the study of limits later in the course. We also wish to foster students' creativity by introducing an inquiry-based learning (IBL) activity. The zipline has them thinking and interacting in the classroom on the first day of class.

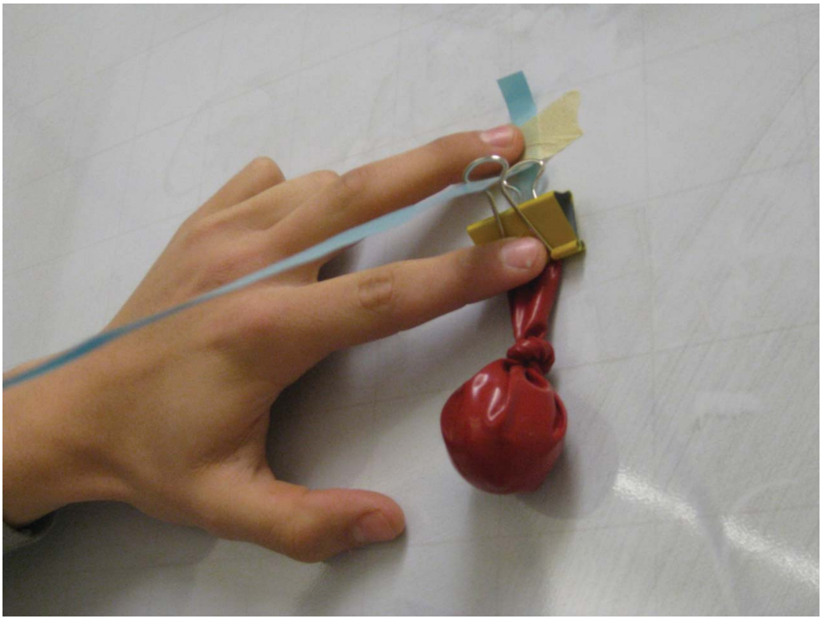

Figure 2. A zipliner (binder clip with a weight) is in position to be released on the zipline (smooth gift wrapping ribbon) (color figure available online).

3. SET-UP AND MATERIALS

To set up the activity, we divide the class into groups of three or four and give each group the following supplies (which can be found at local dollar or discount stores).

1. Zipliner: heavy keychain or one with a weighted object such as golf ball (duct tape works great!), a balloon with a weighted ball inside attached to a binder clip (Figure 2), a small padlock, or similar object.
2. Tape measure.
3. Length of smooth gift wrapping ribbon (at least 6–8 feet or let students cut their own from a whole spool).
4. Scissors.
5. Tape.
6. Marker that writes on the ribbon.
7. Stopwatch (smart phone apps work fine here).

Students should first build and test the zipline. The ribbon can be attached to walls, desks, chairs, or the floor, but should be taut and should be angled so the zipliner takes at least 2 seconds to travel the entire length. Building and testing should take less than 5 minutes.

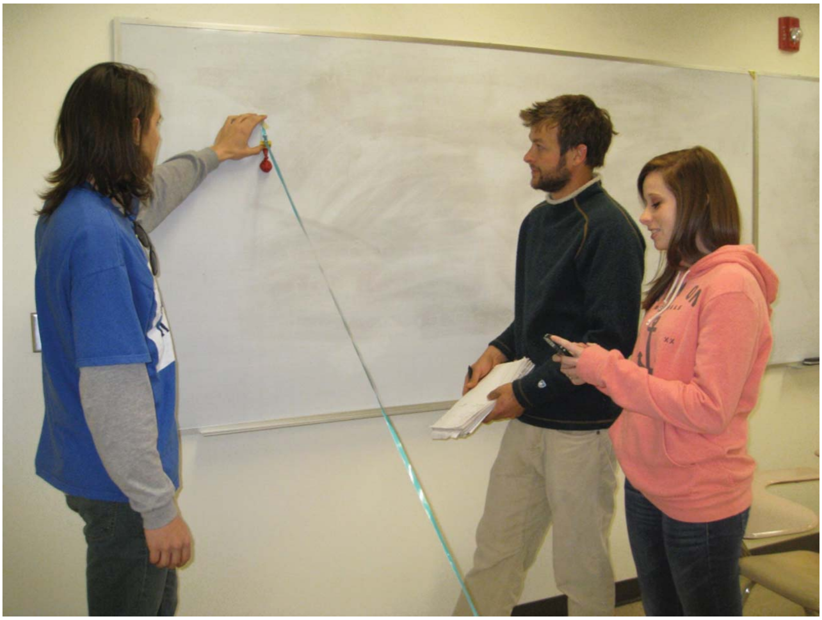

Figure 3. Students record data and collaborate to determine the average velocity of the zipliner (color figure available online).

4. THE ACTIVITY

When the ziplines are built, we ask the students what information they are able to determine with just the materials provided. Ambitious groups calculate angles and slopes, but almost all record time and distance. We keep this exploration part to no more than 5 minutes and then lead into a large class discussion and review of average velocity.

Following this review, students calculate and record the average velocity of the ziplines for the entire trip. (In larger classes, we provide a handout for them to record this information.) This is done by having the students use a stopwatch or stopwatch app to time the zipliner as it travels. In our classes, we have had students start and stop the time when they see the zipliner start and stop, but with advanced smartphones, students could also create movies and use the frame speed to get more accurate results.

We then instruct the students to mark the point on the ribbon one-third of the way from the bottom of the zipline. The students record the average velocity for the top two-thirds of the trip as well as the bottom one-third of the trip and compare these values along with their initial recording of average velocity (Figure 3). Students are often amazed that these values are different! Many have viewed quantities that they can calculate as constant. The idea that velocity can change is eye opening for them. Since calculus is the study of change, this is an important leap! It was not until we used the zipline activity that we saw this pitfall in student reasoning, but now that we have, we better understand some of the struggles our students have with concepts in calculus. We have found our students much more interested in calculating velocity at a particular moment once they realize it may in fact vary.

Once students agree that velocity changes, we begin to talk about instantaneous velocity. We loosely define this as a large group and give an illustrative example. One of our favorite examples comes from asking a commuting student how far she drove to class and how long it took her. We calculate her average velocity and then ask if she traveled exactly that speed the entire time. "Of course not!" she replies. So when we ask how fast she was going at a specific time, the students begin to understand that this is not a trivial question. They are ready to investigate instantaneous velocity.

We present them now with the big question, "Using only the tools you have, can you determine how fast the zipliner is traveling the moment it passes the two-third mark on your ribbon?" The first time we used this activity we asked them to find the instantaneous velocity, but students were not ready to take this step. The beauty of phrasing the question as, "Is it possible . . .," is that both yes and no answers can steer the class towards a discussion of limits. We are careful not to answer this question just yet, but instead leave the students to investigate for the remainder of the class time. Some students argue that it is not possible because they will always need an interval, and others argue that they can get close enough by looking at a much smaller interval. Near the end of class, we ask them to write a "blog" for the next class summarizing the activity and discussing "the big question." (We have not had the students post their blogs online, but we have had much more success phrasing this as a blog-style writing assignment than asking for written summaries of the activity and questions.) The entire activity can be completed in a 50 minute class period with a discussion the following class period.

5. STUDENT HANDOUT

Part I. Construction

Use the materials in your kit to construct a keychain zipline out of ribbon. Run a few test runs and then secure the ends with tape. The keychain should glide quickly and easily down the zipline, but not too fast! (You will be timing intervals later.)

Part II. Average Velocity

Record the length of your zipline. Be sure to use appropriate units!

Length: ______________________________

Record how long it takes the keychain to travel the entire zipline. Use seconds (and fractions of seconds) for your units.

Time: ______________________________

The average velocity of the keychain during this trip can be found using the formula

$$\text{average velocity} = \frac{\text{distance traveled}}{\text{time to travel}}.$$

Find the average velocity. What are the units?
Average Velocity: ______________________________

Part III. Which end is faster? Find the point on the ribbon that is 1/3 of the distance from the bottom of the zipline. Mark this with your marker on the ribbon. You now will find the average velocity for the keychain on the top 2/3 of its trip and on the bottom 1/3. What will you measure in order to do this? Record and explain your calculations below.
Average Velocity for top 2/3: ______________________________
Average Velocity for bottom 1/3: ______________________________
Were these the same? How do they compare to each other and to the average velocity of the total trip you found in Part II?

Part IV. Instantaneous Velocity
We have been able to measure average velocities by calculating distance traveled and the time it took to travel that distance. Is it possible to know exactly how fast the keychain is traveling the moment it passes the 2/3 mark? If so, find this instantaneous velocity. If not, explain why not.

Writing Assignment Prepare a blog entry discussing today's activity. Summarize your group's findings and discuss the question in Part IV. Please type, print and bring your blog to the next class.

6. OUTCOMES AND STUDENT RESPONSES

In their written responses, students are consistently able to summarize the activity and review average velocity correctly. Their responses show that they are pondering the idea of instantaneous velocity as a limit but they are often not able to articulate this until the follow up discussion. In the future, we plan to give students more opportunity to investigate by asking them to calculate the average velocities on multiple intervals around the two-third mark or by asking them to calculate the instantaneous velocity just before the end of the string. Below are selected student responses:

- We have to compute an average velocity over an incredibly small time interval that is not accurately measurable with our class setup.
- . . . it is actually impossible to calculate instantaneous velocity, because... you need intervals for average velocity but we are not trying to get average around a certain point and you can get closer and closer and closer to the right answer by narrowing down the intervals.
- Anything we came up with was always an average which is not instantaneous.

Not all students gave insightful answers, of course, but we have found this activity engages almost all students in thinking more seriously about the question of instantaneous velocity, especially since it is not immediately answered, as it would be in a lecture. We have also found that using a tactile activity gives students permission to explore and think more freely since they do not have the preconceived notion that there is a right answer.

In addition to increasing students' understanding of instantaneous velocity, we have found that using the zipline activity on the first day of class has increased student participation throughout the semester. Students were more open to inquiry-based activities in other units (though we did not teach a strictly IBL calculus course) and more thoughtful in other investigative writing assignments. The student response has been overwhelmingly positive, with some even mentioning the activity in end-of-course evaluations. We've selected a few responses from recent blog assignments below:

- . . . you're forced to think on the first day of college.
- I woke up thinking 'I have calculus. Drat.'Instead, I found myself, not doing problems already written on a newly cleaned chalkboard, but rather, an activity which in real life, could be used to make one incredible ride. Long live the zip line!
- Today was actually a lot of fun and I am excited once again to be in a math class!!!
- If that's what the first class was like, I'm excited for the rest of the semester!

7. FOLLOW UP ACTIVITIES

There are many options to follow up this activity. We have found that a class discussion on instantaneous velocity as the limit of average velocities is well received the class after this activity. We might ask students to share their individual blog responses and build on those or open up the discussion again for a few minutes. We have found that it is essential in these discussions to never use the word velocity alone. Our students can now differentiate between average and instantaneous and we support this by using those adjectives to clarify whichever we are discussing. We have also graphed keychain position versus time and discussed graphical representations of average and instantaneous velocity as secant and tangent line slopes. The zipline activity gives the students a much better framework for these discussions, and we see improved conceptual understanding of limits, velocity, and derivatives.

An alternate extension activity, which we have not yet tested, is to build a giant zipline as a class that would take at least 15 seconds to travel. This would allow the class to accurately measure even more intervals than on their individual ziplines, but of course requires more set up and materials.

As the semester progressed, we often cited the activity when introducing new topics, which provided a visual reference for the students. For example, we related the notion of Δx approaching zero in the limit of a Riemann sum to the parallel procedures in the zipline activity. In addition, we referred to the distance traveled by the zipliner when determining net change. Having the zipline activity as a reference helped students connect different concepts throughout calculus.

8. CONCLUSIONS

The keychain zipline has been a fun and instructive way to start a Calculus I course, and we have found it gives students an increased understanding of instantaneous velocity. In order to do this activity right away, we have redesigned our syllabi to skip the traditional review of functions, incorporating the review later as needed. Students are excited about studying calculus, comfortable in an interactive environment, and curious about future topics.

ACKNOWLEDGEMENT

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BIOGRAPHICAL SKETCHES

Lisa Driskell is an Assistant Professor of Mathematics at Colorado Mesa University. She teaches calculus and several other math courses. She is the advisor to the CMU Math Club, which plans Math Extravaganza, a day of hands-on math activities for high school students. She received her Ph.D. from Purdue University in 2010.

Audrey Malagon received her Ph.D. from Emory University in 2009 and is an Assistant Professor of Mathematics at Virginia Wesleyan College. She teaches a variety of math courses including calculus and incorporates IBL techniques whenever possible. She also advises the student math club and hosts Monday Night Math, a collaborative study time for students and faculty.

Lisa and Audrey have been exploring mathematics together since they were roommates at the Grand Valley State University REU program in 2002. They are both Project NExT fellows.